UEQ #1: How can the rotational motion be described in a measurable and quantitative way?

Unit 8
Rotational Motion

UEQ: How can rotational motion be described, measured, and quantified?

VOCABULARY

- **Radians**
- **Angular Displacement**
- **Angular Velocity**
- **Angular Acceleration**
- **Frequency**
- **Period**
- **Axis of Rotation**
- **Moment Arm**
- **Torque**
- **Right-Hand Rule**
- **Static Equilibrium**
- **Cantilever**
- **Stable Equilibrium**
- **Unstable Equilibrium**
- **Neutral Equilibrium**
- **Moment of Inertia**
- **Rotational Inertia**
- **Rotational Kinetic Energy**
- **Angular Momentum**

UEQ: How can rotational motion be described, measured, and quantified?

LESSON 5

Rotational Inertia
(aka Moment of Inertia)

UEQ: How can rotational motion be described, measured, and quantified?

TIMED ENDURANCE TRAINING 8.5.0

- **Substitute** $a = r\alpha$ **directly into:**

$$F = ma$$

- **Solution:**

$$\boxed{F = mr\alpha}$$

UEQ: How can rotational motion be described, measured, and quantified?

TIMED ENDURANCE TRAINING 8.5.1

- **Substitute** $F = mr\alpha$ **directly into:**

$$\tau = rF$$

- **Solution:**

$$\boxed{\tau = mr^2\alpha}$$

UEQ: How can rotational motion be described, measured, and quantified?

TIMED ENDURANCE TRAINING 8.5.2

- **Substitute** $I = mr^2$ **directly into:**

$$\tau = mr^2\alpha$$

- **Solution:**

$$\boxed{\tau = I\alpha}$$ ← **IMPORTANT: WRITE THIS DOWN!**

UEQ: How can rotational motion be described, measured, and quantified?

TIMED ENDURANCE TRAINING 8.5.3

- **Solve the following equation for α:**

$$\tau = I\alpha$$

- **Solution:**

$$\boxed{\alpha = \frac{\tau}{I}}$$

← **IMPORTANT: WRITE THIS DOWN!**

UEQ: How can rotational motion be described, measured, and quantified?

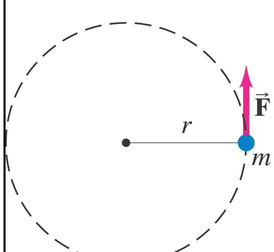
ROETATIONAL MOTION: Moments of Inertia (Rotational Inertia)

- **EQ: What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia?**
- **START: What is easier to start spinning, a hollow ball or a solid ball of the same mass?**

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

8-5 Rotational Dynamics; Torque and Rotational Inertia

Knowing that $F = ma$, we see that $\tau = mr^2\alpha$ (8-11)



This is for a single point mass; what about an **extended** object?

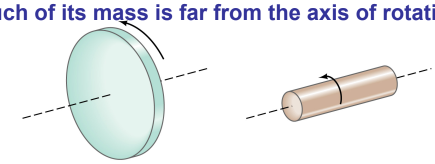
As the angular acceleration is the same for the whole object, we can write:

$$\Sigma\tau = (\Sigma mr^2)\alpha \quad \text{(8-12)}$$

8-5 Rotational Dynamics; Torque and Rotational Inertia

The quantity $I = \Sigma mr^2$ is called the **rotational inertia** (units: kg•m²) of an object.

The **distribution** of mass matters here – these two objects have the same mass, but the one on the left has a greater **rotational inertia**, as so much of its mass is far from the axis of rotation.



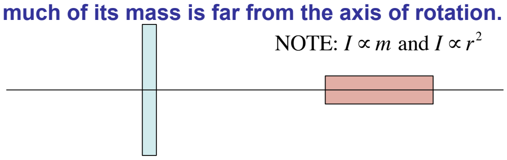
What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

8-5 Rotational Dynamics; Torque and Rotational Inertia

The quantity $I = \Sigma mr^2$ is called the **rotational inertia** (units: kg•m²) of an object.

The **distribution** of mass matters here – these two objects have the same mass, but the one on the left has a greater **rotational inertia**, as so much of its mass is far from the axis of rotation.

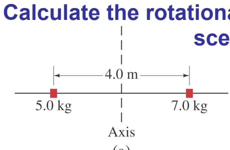
NOTE: $I \propto m$ and $I \propto r^2$



What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

PRACTICE PROBLEM

Calculate the rotational inertia for the following scenarios:

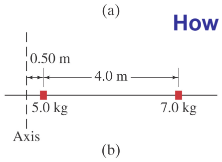


$I = \Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2$

$I_1 = (5.0\text{ kg})(2.0\text{ m})^2 + (7.0\text{ kg})(2.0\text{ m})^2$

$\boxed{I_1 = 48\text{ kg} \bullet \text{m}^2}$

How do these values compare?

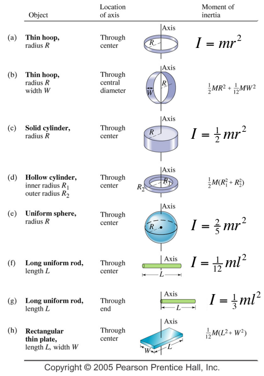


$I = \Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2$ $I_2 \approx 3I_1$

$I_2 = (5.0\text{ kg})(0.5\text{ m})^2 + (7.0\text{ kg})(4.5\text{ m})^2$

$\boxed{I_2 = 143\text{ kg} \bullet \text{m}^2}$

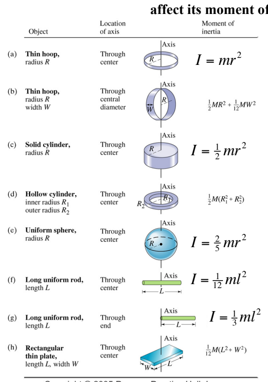
What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?



8-5 Rotational Dynamics; Torque and Rotational Inertia

The **rotational inertia** of an object depends not only on its **mass distribution** but also the location of the **axis** of rotation – compare (f) and (g), for example.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

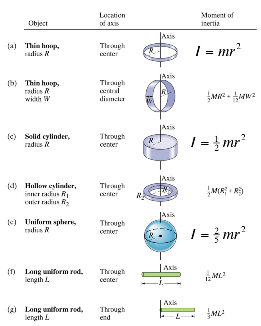


8-5 Rotational Dynamics; Torque and Rotational Inertia

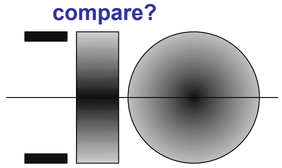
rotational inertia of an object depends not only on its **mass distribution** but also the location of the **axis** of rotation – compare (f) and (g), for example.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THOUGHT QUESTIONS

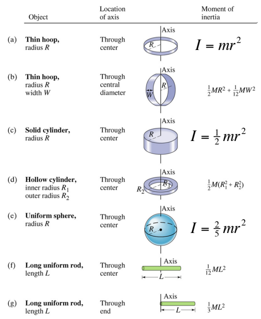


1. Compare the hoop, the solid cylinder, and uniform sphere. Looking head-on, how do the mass distributions compare?



What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THOUGHT QUESTIONS



2. Compare the hoop, the solid cylinder, and uniform sphere. What does each formula have in common?

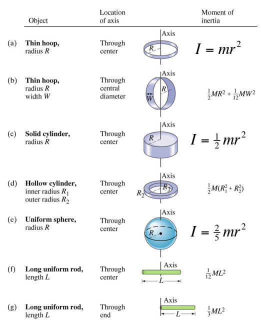
$$\boxed{mr^2}$$

3. How can we generalize this relationship?

$$\boxed{I = nmr^2}$$

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THOUGHT QUESTIONS



4. *Compare the hoop, the solid cylinder, and uniform sphere. Which accelerates faster with the same amount of torque acting on it ($\tau_1=\tau_2$)? Prove it mathematically.

$$\tau_1 = \tau_2$$

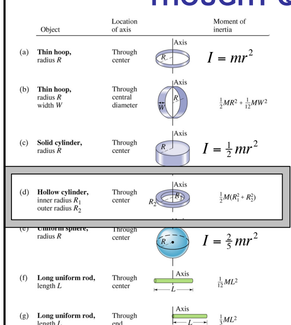
$$mr^2\alpha_1 = \tfrac{1}{2}mr^2\alpha_2$$

$$\boxed{\alpha_2 = 2\alpha_1}$$

*For equal *m* and *r*.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THOUGHT QUESTIONS



4. *Compare the hoop, the solid cylinder, and uniform sphere. Which accelerates faster with the same amount of torque acting on it ($\tau_1=\tau_2$)? Prove it mathematically.

$$\tau_1 = \tau_3$$

$$mr^2\alpha_1 = \tfrac{2}{5}mr^2\alpha_3$$

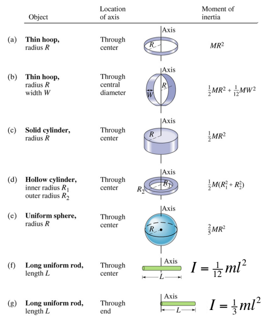
$$\boxed{\alpha_3 = \tfrac{5}{2}\alpha_1}$$

5. Which example applies to toilet paper rolls . . . ?

*For equal *m* and *r*.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THOUGHT QUESTIONS



6. *Compare the long uniform rods (f and g). Which is easier to rotate ($\alpha_1=\alpha_2$)? Prove it mathematically.

$$\tau = mr^2\alpha \Rightarrow \alpha = \frac{\tau}{I}$$

$$\alpha_f = \alpha_g$$

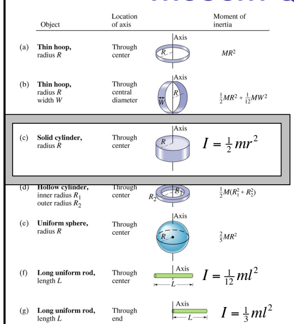
$$\frac{\tau_f}{\frac{1}{12}mr^2} = \frac{\tau_g}{\frac{1}{3}mr^2}$$

$$\boxed{\tau_g = 4\tau_f}$$

*For equal *m* and *r*.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THOUGHT QUESTIONS



6. *Compare the long uniform rods (f and g). Which is easier to rotate ($\alpha_f=\alpha_g$)? Prove it mathematically.

$$\tau = mr^2\alpha \Rightarrow \alpha = \frac{\tau}{I}$$

7. Which example describes the rotational inertia if it is rotated *long* ways?

$$\alpha_1 = \alpha_3$$

$$\frac{\tau_f}{\frac{1}{12}ml^2} = \frac{\tau_g}{\frac{1}{3}ml^2}$$

$$\boxed{\tau_g = 4\tau_f}$$

*For equal *m* and *r*.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

8-6 Solving Problems in Rotational Dynamics

1. **Draw** a diagram.
2. Decide what the **system** comprises.
3. Draw a **free-body diagram** for each object under consideration, including all the **forces** acting on it and where they act.
4. Find the **axis of rotation**; calculate the **torques** around it.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

8-6 Solving Problems in Rotational Dynamics

5. Apply Newton's second law for **rotation**. If the **rotational inertia** is not provided, you need to find it **before** proceeding with this step.
6. Apply Newton's second law for **translation** and other laws and principles as needed.
7. **Solve**.
8. **Check** your answer for units and correct order of magnitude.

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

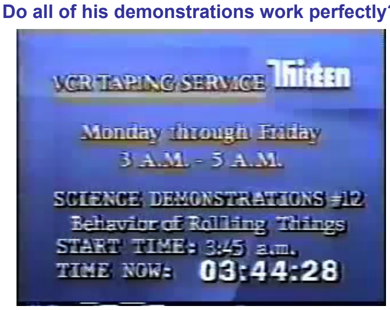
THE AMAZING JULIUS SUMNER MILLER

Write down 5 things you learn in this video!

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THE AMAZING JULIUS SUMNER MILLER

Do all of his demonstrations work perfectly?



What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

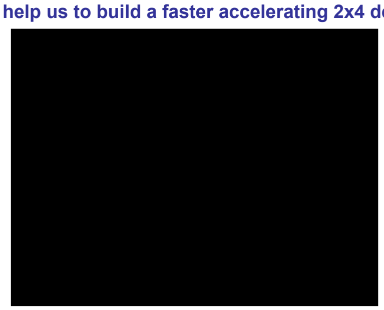
THE AMAZING JULIUS SUMNER MILLER

Has someone modified his demonstration materials?

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

THE AMAZING JULIUS SUMNER MILLER

Can this help us to build a faster accelerating 2x4 derby car?



What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

2x4 Derby Competition

- Important Criteria:
  - Bearings are <u>NOT</u> Allowed
  - Maximum Mass: 1050 g
  - Maximum L x W x H Dimensions: 12" x 8" x 8" (20.35 cm x 15.25 cm x 7.65 cm)
  - The Wheels <u>must</u> be altered from their original design to increase the angular acceleration (and to beat Mr. Roe's modified car).

- You are <u>allowed</u> to modify the wheels, axles, and body. In fact, it is expected!
- SAFETY: Goggles <u>must</u> be worn!

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

2x4 DERBY REPORT

•*One Report for each group of <u>no more than</u> 4 people.
•Refer to the Laboratory Report Expectations Handout.

UEQ(s): †How can rotational motion be described in a measurable and quantitative way?; †What factors affect changes in an object's rotational motion?; †How does an object's mass distribution, interaction with other objects, and force at a distance influence the object's motion?

2x4 Derby RACE DAY

- Competition Grading Criteria:
  - Passing Score (70%): Your car must beat Mr. Roe's unmodified car's time
  - A (93% – 100%): Your Wheels <u>must</u> be altered from their original design to increase the angular acceleration (and it must beat Mr. Roe's modified car's time).
- Report Grading Criteria:
  - Refer to the Handout for the Facets of Understanding Criteria
- SAFETY: Goggles <u>must</u> be worn!

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

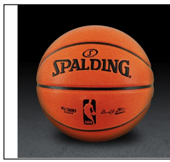
WebAssign/Lab Time

- Work on WebAssign PIM Ch.08E – Rotational Inertia Problems or the 2x4 Derby Lab Report
- Final Copy Criteria
  - State the problem (Ex. Find displacement)
  - Draw a picture/diagram
  - Provide a list or table of all given data (Ex. $\Delta t = 2$ s)
  - Solve the problem symbolically (Ex. $v=\Delta x/\Delta t \rightarrow \Delta x = v\Delta t$)
  - Plug in numbers and units to obtain answer. (Ex. $\Delta x = (5\text{ m/s})(2\text{ s}) = 10$ m)
- Notes about WebAssign:
  - Positive vs. negative answers (Try a negative sign)
  - Look at the final unit (hours or minutes or seconds)

What affects the rotational inertia of an object?; How does the geometry of an object affect its moment of inertia of an object?

Summary

- Answer the Essential Questions.
- Ticket out the Door:
  - If all of the masses and radia are the same, rank the following objects in terms of their rotational inertias from the easiest to the hardest to rotate:
    Thin hoop, solid sphere, solid disk, and a hollow sphere ($I = \frac{2}{3} mr^2$)



- HW:
  - WebAssign PIM Ch.08E – Rotational Inertia Problems
  - 2x4 Derby Lab Report (Questions and Conclusions)