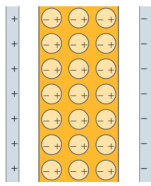
Chapter 24

Capacitance, Dielectrics, Electric Energy Storage



Units of Chapter 24

- Capacitors (1, 2, & 3)
- Determination of Capacitance (4 & 5)
- Capacitors in Series and Parallel (6 & 7)
- Electric Energy Storage (8 - 12, 14, & 15)
- Dielectrics (13)
- Molecular Description of Dielectrics

24.1 Capacitors

A capacitor has the ability to store electric charge and widely used for:

•Storing charge for camera flashes and a backup energy source for computers.

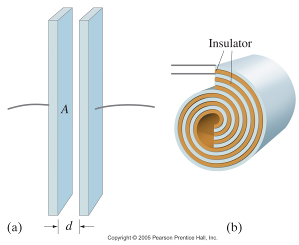
•Protecting circuits by blocking energy surges.

•Tuning Radio frequencies

•Memory of the binary code in RAM

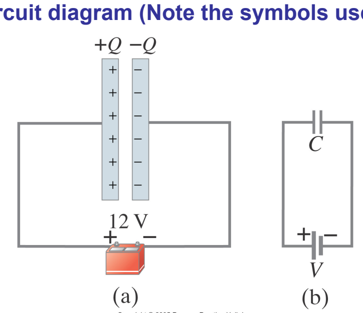
24.1 Capacitors

A capacitor consists of two conductors that are close but not touching.



24.1 Capacitors

Parallel-plate capacitor connected to battery. (b) is a circuit diagram (Note the symbols used).



24.1 Capacitors

When a capacitor is connected to a battery, the charge on its plates is proportional to the voltage:

$$Q = CV_{ba}$$

The quantity C is called the capacitance.

Unit of capacitance: the farad (F)

1 F = 1 C/V

The capacitance of most capacitors are 1 pF - 1 μF

Calculating the Potential Difference, V_{ba}, in a Uniform Field

(1) Because the field and the direction of motion are the same, cos *θ* = 1,

(2) Because E is uniform, it can be pulled out of the integral.

$$V_{ba} = -\int_a^b E \cdot dl = \int_a^b E \cdot dl = \int_a^b E\,dl\cos\theta;$$

$$V_{ba} = -\int_a^b E\,dl\underbrace{\cos(0°)}_{1} = -E\int_a^b dl = -E\int_{a=0}^{b=d} dl$$

$$V_{ba} = \boxed{-Ed}$$

(3) The negative sign implies that the potential is decreasing as it moves a distance, *d*, in the electric field.

24.2 Determination of Capacitance

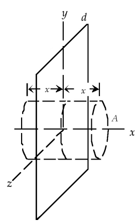
For a parallel-plate capacitor:

Gauss's Law (Cylinder embedded in a plate)

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{encl}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad \left(\text{Note:} \quad \sigma = \frac{Q}{A} \text{ and } Q = \sigma A\right)$$

$$E2\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0} \text{ for each plate}$$

$$E_{net} = \sum E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$



24.1 Capacitors

The capacitance does not depend on the voltage; it is a function of the geometry (size, shape, and relative position of the two conductors) of and material that separates the capacitor.

For a parallel-plate capacitor:

$$C = \epsilon_0 \frac{A}{d}$$

24.2 Determination of Capacitance

For a parallel-plate capacitor:

The relation between electric field and electric potential is given by

$$V_{ba} = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$$

We can take the line integral along a path antiparallel to the field lines, from one plate to the other; then $\theta = 180^\circ$ and $\cos 180^\circ = -1$, so

$$V_{ba} = V_b - V_a = -\int_a^b E\,dl\cos 180^\circ = +\int_a^b E\,dl = \frac{Q}{\varepsilon_0 A}\int_a^b dl = \frac{Qd}{\varepsilon_0 A}$$

This relates Q to V_{ba} , and from it we can get the capacitance C in terms of the geometry of the plates:

$$C = \frac{Q}{V_{ba}} = \frac{\cancel{Q}}{\frac{\cancel{Q}d}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d}$$

24.2 Determination of Capacitance

For a cylindrical capacitor:

Gauss's Law
(Cylinder around a long wire)

$$\oint \mathbf{E}\cdot d\mathbf{A} = EA_{side} + EA_{top} + EA_{bottom} = \frac{Q_{encl}}{\varepsilon_0}$$

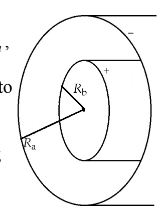
$$EA_{side} + \underbrace{EA_{top}}_{=0} + \underbrace{EA_{bottom}}_{=0} = E(2\pi rL) = \frac{Q_{encl}}{\varepsilon_0}$$

$$E(2\pi rL) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 rL} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

24.2 Determination of Capacitance

For a cylindrical capacitor:

To obtain $C=Q/V_{ba}$, we need to determine the potential difference between the cylinders, V_{ba} , in terms of Q. To obtain V_{ba} in terms of Q, write the line integral from the outer cylinder to the inner one (so $V_{ba} > 0$) along a radial line. [Note that **E** points outward but ***dl*** points inward for our chosen direction of integration; the angle between **E** and ***dl*** is 180° and cos 180° = -1. Also, $dl = -dr$ because dr increases outward. These minus signs cancel.]



24.2 Determination of Capacitance

For a cylindrical capacitor:

$$V_{ba} = -\int_a^b E \cdot dl = -\frac{Q}{2\pi\varepsilon_0 L}\int_{R_a}^{R_b}\frac{dr}{r}$$

$$V_{ba} = -\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_b}{R_a}\right) = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{R_a}{R_b}\right)$$

$$C = \frac{Q}{V_{ba}} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\varepsilon_0 L}\ln\left(\frac{R_a}{R_b}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{R_a}{R_b}\right)}$$

24.2 Determination of Capacitance

Recommended Practice:

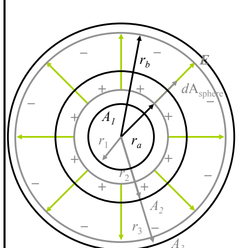
Solve for a Spherical Capacitor

$$C = 4\pi\varepsilon_o\left(\frac{r_a r_b}{r_a - r_b}\right)$$

Recommended Application:

•Example 24.1 on page 615

Concentric Conducting Spherical Shells



At radius r_3 $(r > r_b)$,

$$\oint \mathbf{E}\cdot d\mathbf{A} = EA_3\cos\theta = EA_3 = E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

Because the enclosed charge **outside** of a conductor is zero, $\boxed{E_{outside} = 0}$

At radius r_2 $(r_a < r < r_b)$,

$$\oint \mathbf{E}\cdot d\mathbf{A} = EA_2\cos\theta = EA_2 = E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

Therefore, $\boxed{E_{between} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1^2}}$

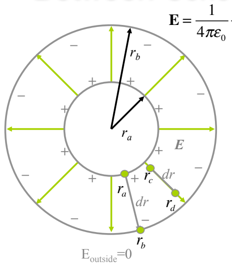
At radius r_1 $(r < r_a)$,

$$\oint \mathbf{E}\cdot d\mathbf{A} = EA_1\cos\theta = EA_2 = E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

Because the enclosed charge **inside** of a conductor is zero, $\boxed{E_{inside} = 0}$

Determination of V from **E**:

Between Concentric Spheres ($r_a < r < r_b$)



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2};\; V_{cd} = -\int_c^d \mathbf{E}\cdot d\mathbf{l} = -\int_{r_c}^{r_d} E\,dr\underbrace{\cos\theta}_{=1} = -\int_{r_c}^{r_d} E\,dr$$

$$V_{cd} = -\int_{r_c}^{r_d}\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\int_{r_c}^{r_d}\frac{1}{r^2}dr$$

$$V_{cd} = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_c}^{r_d} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_c}^{r_d} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_d}-\frac{1}{r_c}\right)$$

$$\boxed{V_{cd} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_c - r_d}{r_c r_d}\right)} \text{ at } r_a < r_c < r_d < r_b$$

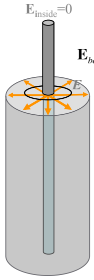
Note: If $r_c \to r_a$ and $r_d \to r_b$, then

$$V_{ba} = \boxed{\frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_a\, r_b}\right)}$$

Determination of V from **E**:

Between Concentric Cylinders ($r_a < r < r_b$)

$E_{inside}=0$



At radius $r_a < r < r_b$,

$$\mathbf{E}_{between} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r}\right)$$

$$V_{cd} = -\int_c^d \mathbf{E}\cdot d\mathbf{l} = -\int_{r_c}^{r_d} E\,dr\underbrace{\cos\theta}_{=1} = -\int_{r_c}^{r_d} E\,dr$$

$$V_{cd} = -\int_{r_c}^{r_d}\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_c}^{r_d}\frac{1}{r}dr$$

$$V_{cd} = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{1}{r}\right)\right]_{r_c}^{r_d} = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{1}{r}\right)\right]_{r_c}^{r_d} = \frac{\lambda}{2\pi\varepsilon_0}\left(\ln(r_d) - \ln(r_c)\right)$$

$E_{outside}=0$

$$V_{cd} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_d}{r_c}\right)} \text{ at } r_a < r_c < r_d < r_b$$

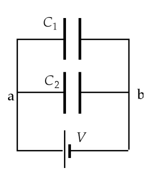
Note: If $r_c \to r_a$ and $r_d \to r_b$, then

$$V_{ba} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_b}{r_a}\right)}$$

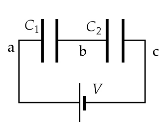
24.3 Capacitors in Series and Parallel

Capacitors can be connected in two ways:

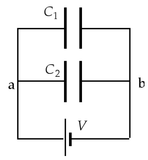
•In Parallel



•In Series



24.3 Capacitors in Parallel



When a battery of voltage V is connected such that all of the left hand plates reach the same potential V_a and all of the right hand plates reach the same potential V_b.

24.3 Capacitors in Parallel

Each capacitor plate acquires a charge given by:

$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$

The total charge Q that must leave the battery is then:

$$Q = Q_1 + Q_2 = C_1 V + C_2 V$$

Finding a single equivalent capacitor that will hold the same charge Q at the same voltage V gives:

$$Q = C_{eq} V$$

24.3 Capacitors in Parallel

Finding a single equivalent capacitor that will hold the same charge Q at the same voltage V gives:

$$Q = C_{eq} V$$

Combining the above equation with the previous equation gives:

$$C_{eq} V = C_1 V + C_2 V = (C_1 + C_2) V$$

or

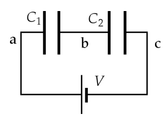
$$C_{eq} = C_1 + C_2$$

24.3 Capacitors in Parallel

The net effect of connecting capacitors in parallel increases the capacitance because we are increasing the area of the plates where the charge can accumulate:

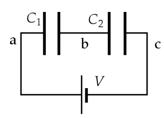
$$C = \epsilon_0 \frac{A}{d}$$

24.3 Capacitors in Series



When a battery of voltage *V* is connected to capacitors that are connected end to end. A charge +*Q* flows from the battery to one plate of C_1, and -*Q* flows to one plate of C_2. The region *b* was originally neutral; so the net charge must still be zero.

24.3 Capacitors in Series



The +*Q* on the left plate of C_1 attracts a charge of -*Q* on the opposite plate. Because region *b* must have a zero net charge, there is thus a +*Q* on the left plate of C_2.

24.3 Capacitors in Series

A single capacitor that could replace these two in series without affecting the circuit (*Q* and *V* stay the same) would have a capacitance C_{eq} where:

$$Q = C_{eq}V \Rightarrow V = Q/C_{eq}$$

The total voltage *V* across the two capacitors in series must equal the sum of the voltages across each capacitor

$$V = V_1 + V_2$$

24.3 Capacitors in Series

Because each capacitor plate acquires a charge given by:

$$Q = C_1V_1 \Rightarrow V_1 = Q/C_1$$
$$Q = C_2V_2 \Rightarrow V_2 = Q/C_2$$

24.3 Capacitors in Series

Solving each for *V* and combining the previous equations gives:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Note that the equivalent capacitance C_{eq} is smaller than the smallest contributing capacitance.

24.3 Capacitors in Series and Parallel

Application:

Determine the capacitance, C_{eq}, of a single capacitor with the same effect as the 4 capacitors combined in series-parallel.

(Let $C_1 = C_2 = C_3 = C_4 = C$.)

24.3 Capacitors in Series and Parallel

Application (con't):

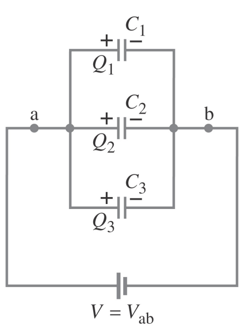
Determine the capacitance, C_{eq}, of a single capacitor with the same effect as the 4 capacitors combined in series-parallel.

(Let $C_1 = C_2 = C_3 = C_4 = C$.)

Determine the charge on each capacitor and potential difference across each if the capacitors were charged by a 12-V battery.

24.3 Capacitors in Series and in Parallel

Capacitors in parallel have the same voltage across each one:



24.3 Capacitors in Series and in Parallel

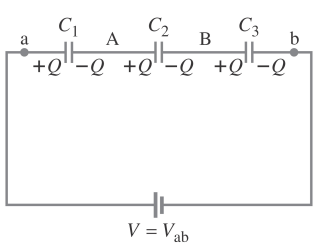
In this case, the total capacitance is the sum:

$$C_{eq}V = C_1V + C_2V + C_3V = (C_1 + C_2 + C_3)V$$

$$C_{eq} = C_1 + C_2 + C_3 \qquad \text{[parallel] (24-3)}$$

24.3 Capacitors in Series and in Parallel

Capacitors in series have the same charge:



24.3 Capacitors in *Series* and in Parallel

In this case, the reciprocals of the capacitances add to give the reciprocal of the equivalent capacitance:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \text{[series] (24-4)}$$

24.4 Electric Energy Storage

A charged capacitor stores electric energy; the energy stored is equal to the work done to charge the capacitor. The net effect of charging a capacitor is to remove charge from one plate and add it to the other plate. This is what the battery does when it is connected to a capacitor. A capacitor does not become charged instantly. It takes time. Initially, when a capacitor is uncharged, it requires no work to move the first bit of charge over.

24.4 Electric Energy Storage

When some charge is on each plate, it requires work to add more charge of the same sign because of electric repulsion. The more charge already on the plate, the more work is required to add additional charge.

24.4 Electric Energy Storage

The work needed to add a small amount of charge dq, when a potential difference V is across the plates is $dW = V\,dq$. Since $V=q/C$ at any moment where C is the capacitance, the work needed to store a total charge Q is

$$W = \int_0^Q V dq = \frac{1}{C}\int_0^Q q dq = \frac{1}{2}\frac{Q^2}{C}$$

24.4 Electric Energy Storage

The energy "stored" in a capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C}$$

When the capacitor C carries charges $+Q$ and $-Q$ on its two conductors. Since $Q=CV$ (and $V=Q/C$), where V is the potential difference across the capacitor, we can also write

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

24.4 Electric Energy Storage

It is useful to think of the energy stored in a capacitor as being stored in the electric field between the plates. Using

$$U = \frac{1}{2}CV^2,\ C = \varepsilon_o \frac{A}{d},\ \text{and } V = Ed$$

for a parallel plate capacitor, solve the energy per volume as a function of the electric field:

$$\frac{U}{Volume}(E) =$$

24.4 Electric Energy Storage

The energy density, defined as the energy per unit volume, is the same no matter the origin of the electric field:

$$energy\ density = u = \frac{U}{Volume} = \frac{1}{2}\varepsilon_o E^2$$

The sudden discharge of electric energy can be harmful or fatal. Capacitors can retain their charge indefinitely even when disconnected from a voltage source – be careful!

24.4 Storage of Electric Energy

Heart defibrillators use electric discharge to "jump-start" the heart, and can save lives.

24.5 Dielectrics

A dielectric is an insulator that is placed between two capacitor plates.

24.5 The Purpose of Dielectrics

1. They do not allow charge to flow between them as easily as in air. The result: higher voltages can be applied without charge passing through the gap
2. They allow the plates to be placed closer together without touching. The result: the capacitance is increased because *d* is less.

24.5 The Purpose of Dielectrics

3. If the dielectric fills the space between the two conductors, it increases the capacitance by a factor of *K*, the dielectric constant.

$$C = KC_0$$

where C_0 is the capacitance when the space between the two conductors of the capacitor is a vacuum and C is the capacitance when the space is filled a material whose dielectric constant is K.

TABLE 24-3 Dielectric constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Rubber, neoprene	6.7	12×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Dielectrics

Dielectric strength is the maximum field a dielectric can experience without breaking down.

Note the similarity between a vacuum and air.

24.5 Dielectrics

Capacitance of a parallel-plate capacitor filled with dielectric:

$$C = K\varepsilon_o \frac{A}{d}$$

Because the quantity $K\varepsilon_o$ appears so often in formulas, a new quantity known as the permittivity of the material is defined as

$$\varepsilon = K\varepsilon_o$$

24.5 Dielectrics

Therefore the capacitance of a parallel-plate capacitor becomes

$$C = \varepsilon \frac{A}{d}$$

The energy density stored in an electric field *E* in a dielectric is given by

$$u = \frac{1}{2} K \varepsilon_o E^2 = \frac{1}{2} \varepsilon E^2$$

24.5 Dielectrics

Application #1:

A parallel-plate capacitor, filled with a dielectric with $K = 2.2$, is connected to a 12 V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A = 2.0\ \text{m}^2$, and are separated by $d = 4.0$ mm. Find the (a) capacitance, (b) charge on the capacitor, (c) electric field strength, and (d) energy stored in the capacitor.

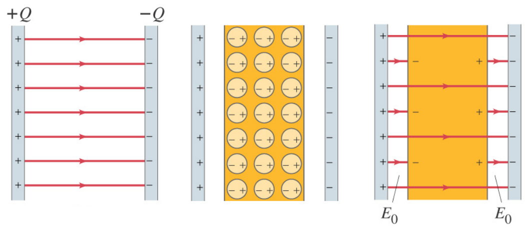
24.5 Dielectrics

Application #2:

The dielectric from the previous parallel-plate capacitor is carefully removed, without changing the plated separation nor does any charge leave the capacitor. Find the new values of (a) capacitance, (b) the charge on the capacitor, (c) the electric field strength, and (d) the energy stored in the capacitor.

Molecular Description of Dielectrics

The molecules in a dielectric tend to become oriented in a way that reduces the external field.



Molecular Description of Dielectrics

This means that the electric field within the dielectric is less than it would be in air, allowing more charge to be stored for the same potential.

Partial Summary of Chapter 24

- Capacitors in parallel:

$$C_{eq} = C_1 + C_2 + C_3$$

- Capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$