Chapter 28

Sources of Magnetism

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- 28.2 Force between Two Parallel Wires: 1 - 8
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- †28.9 Magnetic Fields in Magnetic Materials; Hysteresis
- †28.10 Paramagnetism and Diamagnetism

28.1 Magnetic Field Due to a Long Straight Wire

The field is inversely proportional to the distance from the wire:

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

The constant μ_0 is called the permeability of free space, and has the value:

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$$

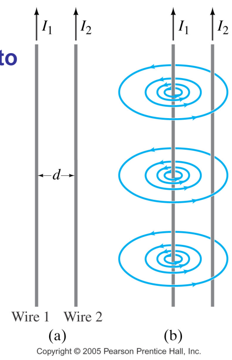
28.2 Force between Two Parallel Wires

The magnetic field produced at the position of wire 2 due to the current in wire 1 is:

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}$$

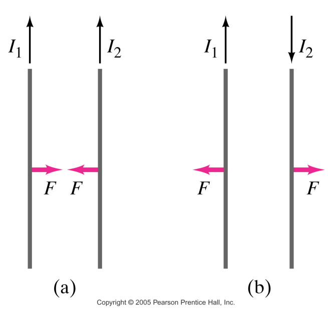
The force this field exerts on a length l_2 of wire 2 is:

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l_2$$



28.2 Force between Two Parallel Wires

Parallel currents attract; antiparallel currents repel.

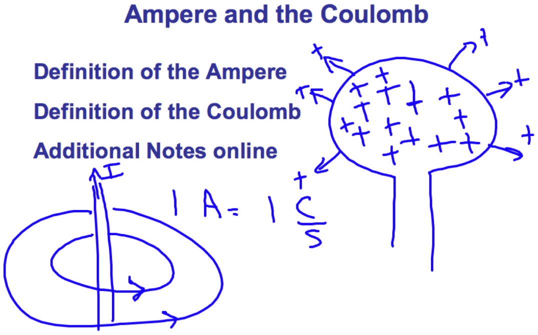


28.4 Operational Definitions of the Ampere and the Coulomb

Definition of the Ampere

Definition of the Coulomb

Additional Notes online



$1\ A = 1\ \frac{C}{s}$

28.4 Operational Definitions of the Ampere and the Coulomb

Definition of the Ampere

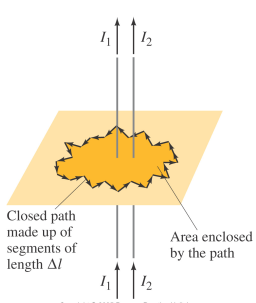
Definition of the Coulomb

Additional Notes online

28.4 Ampère's Law

Ampère's law relates the magnetic field around a closed loop toot the total current flowing through the loop.

$$\Sigma B_{\parallel}\,\Delta l = \mu_0 I_{\mathrm{encl}}$$



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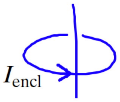
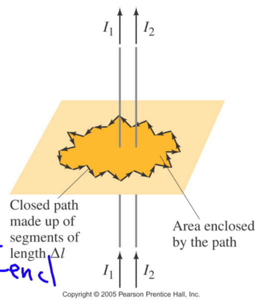
$$\Sigma B_{\parallel}\,\Delta l = \mu_0 I_{\text{encl}}$$



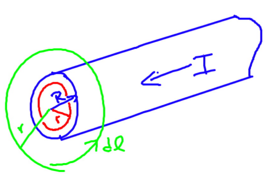


$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$B(2\pi r) = \mu_0 I_{encl}$$

$$F = I l B \qquad B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$\frac{F}{l} = IB = \frac{\mu_0}{2\pi}\frac{I_1}{r} I_2$$

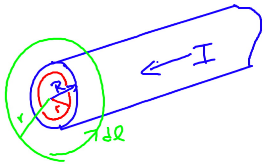


a) $r > R$

$$\oint B\cdot dl = \mu_0 I_{encl}$$

$$B(2\pi r) = \mu_0 I_{encl}$$

$$B = \frac{\mu_0}{2\pi}\frac{I_{encl}}{r}$$

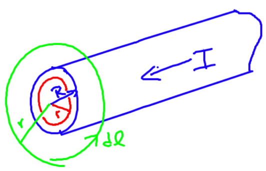


b) $r < R$ $\quad I_{encl} = I_{tot} \dfrac{\cancel{\pi} r^2}{\cancel{\pi} R^2}$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B(2\pi r) = \mu_0 I_{tot} \frac{r^2}{R^2}$$

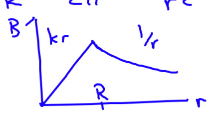
$$B = \frac{\mu_0}{2\pi} \frac{I_{tot}\, r}{R^2}$$



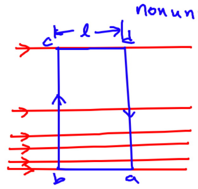
c) $r = R$

$$B = \frac{\mu_0}{2\pi} I_{tot} \frac{r}{R^2} = \frac{\mu_0}{2\pi} I_{tot} \frac{r}{r^2}$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$



$\vec{B}$ cannot be unidirectional and nonuniform

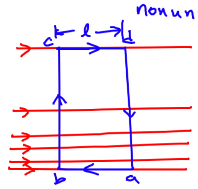


$$|B_{ab}| > |B_{cd}|$$

$$\oint B \cdot dA = \mu_0 \cancel{I_{encl}} \rightarrow 0$$

Does not happen

$\vec{B}$ cannot be unidirectional and nonuniform



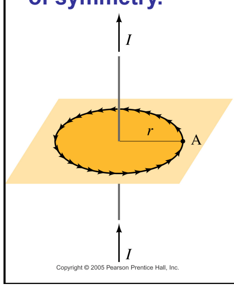
$|B_{ab}| > |B_{cd}|$

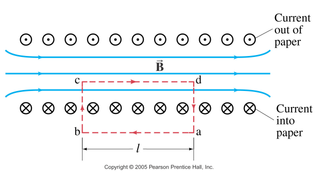
$\oint B \cdot dA = \mu_0 \cancel{I_{encl}}^{0}$

Does not happen

28.4 Ampère's Law

Ampère's law can be used to calculate the magnetic field in situations with a high degree of symmetry.

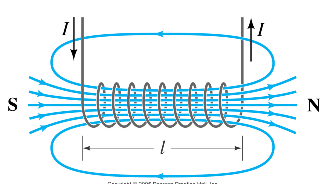




28.5 Magnetic Field of a Solenoid and Toroid

A solenoid is a long coil of wire. If it is tightly wrapped, the magnetic field in its interior is almost uniform:

$$B = \mu_0 I N / l$$



Solenoids

$$\oint B\cdot dl = \int_a^b B\cdot dl + \int_b^c \cancel{B\cdot dl}^{\,0} + \int_c^d B\cdot dl + \int_d^a \cancel{B\cdot dl}^{\,0}$$

$B \to 0$ $\cos 90° = 0$ $\cos 90° = 0$

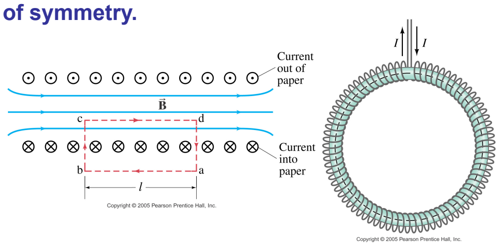
$$\oint B\cdot dl = \int_c^d B\cdot dl = Bl = \mu_0 I_{encl}$$

$$Bl = \mu_0 N I \Rightarrow B = \mu_0 \frac{N}{l} I = \mu_0 n I$$

IMPORTANT NOTE: *n* is equal to the number of loops per unit length (aka. Loop Density). It is NOT the number of wires!

28.5 Magnetic Field of a Solenoid and a Toroid

Ampère's law can be used to calculate the magnetic field in situations with a high degree of symmetry.



IMPORTANT NOTE: *n* is equal to the number of coils per unit length (aka. Coil Density). It is NOT the number of wires!

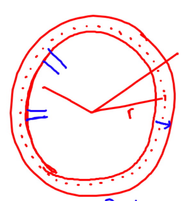
$$\oint B\cdot dl = \mu_0 I_{encl}$$

$$B(2\pi r) = \mu_0 I_{encl}$$

Current enclosed stupid

$$B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0}{2\pi}\frac{N}{r} I$$



If Δr is small then $n = \frac{N}{2\pi r}$

# of inner field lines $\approx$ # of outer field lines

$$\boxed{B = \mu_0 n I}$$

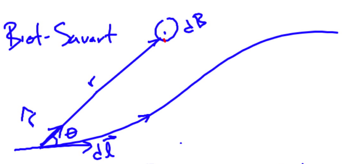
$B = \mu_0 n i$

28.6 Biot-Savart Law

Biot-Savart Law

- ***B*** **due to current *I* in straight wire**
- **Current loop**
- **Magnetic Dipole Moment**
- ***B*** **due to a wire segment**

Biot-Savart

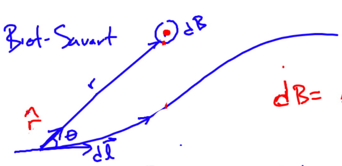


To find the $\vec{B}$ at any point in space it is necessary to include all currents (not just I_{encl} as in Ampère's Law)

Magnitude of B

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dl\,\sin\theta}{r^2} \quad ; \quad B = \int dB$$

Biot-Savart



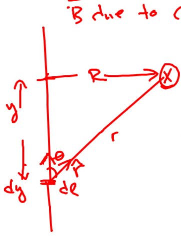
$$dB = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

To find the $\vec{B}$ at any point in space it is necessary to include all currents (not just I_{encl} as in Ampère's Law)

Magnitude of B

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dl\,\sin\theta}{r^2} \quad ; \quad B = \int dB$$

$\vec{B}$ due to current I in a straight wire



Magnitude of B

$$B = \frac{\mu_0 I}{4\pi} \frac{d\ell \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \int_{y=-\infty}^{y=\infty} \frac{dy \sin\theta}{r^2}$$

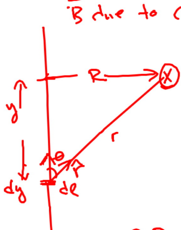
where $dy = d\ell$

$r^2 = y^2 + R^2$

$\sin\theta = \frac{R}{r} = \frac{R}{\sqrt{y^2+R^2}}$

$\vec{B}$ due to current I in a straight wire

(Page 2)



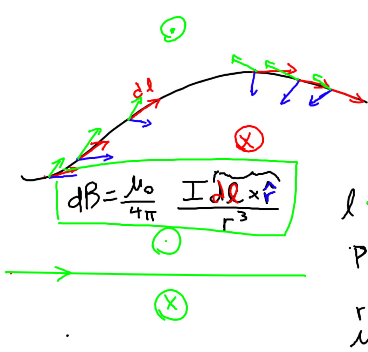
Magnitude of B

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{\frac{R}{r}\, dy}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{R}{r^3}\, dy$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{R}{(y^2+R^2)^{3/2}}\, dy \quad \frac{\mu_0 I R}{4\pi} \left[\frac{y}{R^2\sqrt{y^2+R^2}} \right]_{-\infty}^{\infty}$$

$$= \frac{\mu_0 I R}{4\pi} \frac{2}{R^2} = \boxed{\frac{\mu_0 I}{2\pi R}}$$



$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I\, \overbrace{d\ell \times \hat{r}}}{r^3}}$$

$\ell \sin\theta$

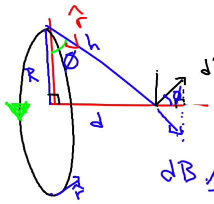
$p \times E$ $\quad pE \sin\theta$

$r \times F$ $\quad rF \sin\theta$

$\mu \times B$ $\quad \mu B \sin\theta$

Biot-Savart

$\vec{B}$ due to a current loop



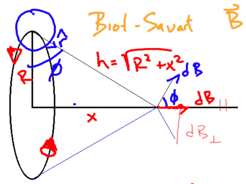
$$dB = \frac{\mu_0 I}{4\pi}\frac{d\ell \times \hat{r}}{r^2}$$

$$\int_0^{2\pi} \sin\phi \to 0$$

$$dB\cos\phi = dB\frac{R}{h}$$

$$dB\ \frac{\mu_0 I}{4\pi}\frac{1}{h^2}\frac{R}{h} \quad d\ell = \frac{\mu_0 I}{4\pi}$$

Biot-Savart $\vec{B}$ due to current loop

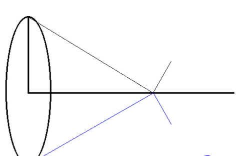


$$d\ell \times \hat{r} = d\ell \sin 90° = d\ell$$

Due to symmetry $\int dB_\perp \to 0$

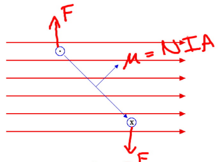
$$dB_{\parallel} = dB\cos\phi = dB\frac{R}{h} = \frac{\mu_0 I}{4\pi\, h^2}\overbrace{d\ell \times \hat{r}}^{d\ell}\left(\frac{R}{h}\right)$$

$$B_{\parallel} = \frac{\mu_0 I}{4\pi h^3} R \int_0^{2\pi R} d\ell = \frac{\mu_0 I R^2}{2 h^3} = \frac{\mu_0 I R^2}{2\,(R^2+x^2)^{3/2}}$$



@ $x=0$

$$\frac{\mu_0 I R^2}{2\,(R^2+\cancel{x^2}^{\,0})^{3/2}} = \boxed{\frac{\mu_0 I}{2R} = B_{max}}$$



Not permeab,,
$\mu =$ * Magnetic dipole moment

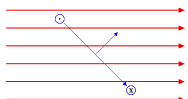
$N =$ # of loops

$I =$ Current

$A =$ Area of loop

$$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 I \overbrace{\pi R^2}^{A}}{2\pi(R^2+x^2)^{3/2}} \rightarrow \frac{\mu_0 \overbrace{NIA}^{\mu}}{2\pi(R^2+x^2)^{3/2}}$$

$$\boxed{B = \frac{\mu_0 \mu}{2\pi(R^2+x^2)^{3/2}}}$$ Magnetic dipole

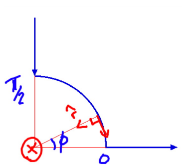


$$B = \frac{\mu_0 \mu}{2\pi(R^2+x^2)^{3/2}}$$

@ $x >> R$ $\quad B = \frac{\mu_0 \mu}{2\pi(x^2)^{3/2}} = \frac{\mu_0 \mu}{2\pi \, x^3}$ (dipole on x-ax

Similar to $E = \frac{1}{4\pi\epsilon_0}\frac{p}{x^3}$

@ $x = 0$ $\quad B = \frac{\mu_0 \mu}{2\pi R^3}$



$$dB = \frac{\mu_0 I}{4\pi r^2} \underbrace{d\vec{l} \times \hat{r}}_{\theta = 90}$$

$$dB = \frac{\mu_0 I}{4\pi r^2} dl$$

$$B = \frac{\mu_0 I}{4\pi R^2}\int_0^{\frac{\pi}{2}R} dl = \frac{\mu_0 I}{4\pi R^2}\left[\frac{\pi}{2}R - 0\right]$$

$$\boxed{B = \frac{\mu_0 I}{8R}}$$

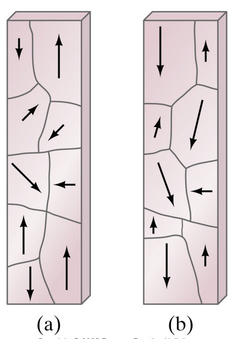
28.7 Magnetic Materials - Ferromagnetism

Ferromagnetic materials are those that can become strongly magnetized, such as iron and nickel.

These materials are made up of tiny regions called domains; the magnetic field in each domain is in a single direction.

28.7 Magnetic Materials - Ferromagnetism

When the material is unmagnetized, the domains are randomly oriented. They can be partially or fully aligned by placing the material in an external magnetic field.



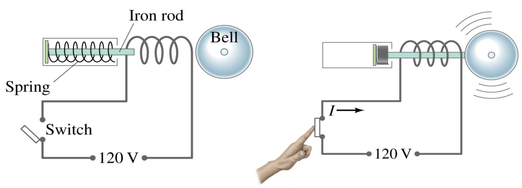
28.7 Magnetic Materials - Ferromagnetism

A magnet, if undisturbed, will tend to retain its magnetism. It can be demagnetized by shock or heat.

The relationship between the external magnetic field and the internal field in a ferromagnet is not simple, as the magnetization can vary.

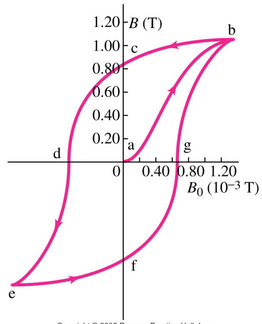
28.8 Solenoids and Electromagnets

If a piece of iron is inserted in the solenoid, the magnetic field greatly increases. Such electromagnets have many practical applications.



28.9 Magnetic Fields in Magnetic Materials; Hysteresis

Starting with unmagnetized material and no magnetic field, the magnetic field can be increased, decreased, reversed, and the cycle repeated. The resulting plot of the total magnetic field within the ferromagnet is called a hysteresis curve.



Summary of Chapter 28

- Magnitude of the field of a long, straight current-carrying wire:

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

- Parallel currents attract; antiparallel currents repel

Summary of Chapter 28

• Magnetic field inside a solenoid:

$$B = \mu_0 I N / l$$

• Ampère's law:

$$\Sigma B_{||} \Delta l = \mu_0 I_{\text{encl}}$$