Chapter 30

Inductance and Electromagnetic Oscillations

Units of Chapter 30

- Faraday's Law: Changing current in a circuit will induce emf in that circuit as well as others nearby
- Self-Inductance: Circuit induces emf in itself
- Mutual Inductance: Circuit induces emf in second circuit

30-1 Mutual Inductance

emf induced in circuit 2 by changing currents in circuit 1, through mutual inductance:

$$\mathscr{E}_{21} = -M\frac{dI_1}{dt}$$

- The mutual inductance *M* depends only on the geometry of the two-circuit system
- subscripts are omitted, as $M_{21} = M_{12}$

Pre Inductance Review

Review the BIG 4 (Gauss^2, Faraday, Amperes)

Determine B for a solenoid

Solve for the current in the solenoid

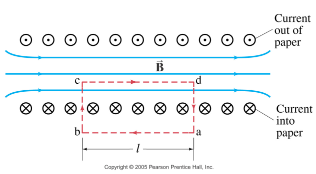
Determine the magnetic flux in the solenoid

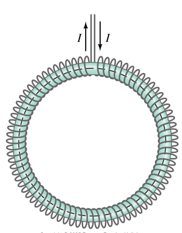
Determine the self inductance

$$1\ \mathrm{H} = 1\ \mathrm{Wb/A} = 1\ \mathrm{T\cdot m^2/A} = 1\ \Omega \bullet \mathrm{s}$$

28.5 Magnetic Field of a Solenoid and a Toroid

Ampère's law can be used to calculate the magnetic field in situations with a high degree of symmetry.





Solenoids

$$\oint B\cdot dl = \underbrace{\int_a^b B\cdot dl}_{B\to 0} + \underbrace{\int_b^c \cancel{B\cdot dl}}_{\cos 90^\circ = 0} + \int_c^d B\cdot dl + \underbrace{\int_d^a \cancel{B\cdot dl}}_{\cos 90^\circ = 0}$$

$$\oint B\cdot dl = \int_c^d B\cdot dl = Bl = \mu_0 I_{encl}$$

$$Bl = \mu_0 N I \Rightarrow B = \mu_0 \frac{N I}{l} = \mu_0 n I$$

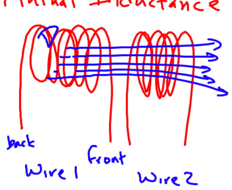
IMPORTANT NOTE: *n* is equal to the number of loops per unit length (aka. Loop Density). It is NOT the number of wires!

30-1 Mutual Inductance

Units of inductance: Henry

$$1\ \mathrm{H} = 1\ \mathrm{Wb/A} = 1\ \mathrm{T\cdot m^2/A} = 1\ \Omega\cdot\mathrm{s}$$

Mutual Inductance



defined as

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt}$$

$$\Phi_{21} = \frac{M_{21} I_1}{N_2} \Rightarrow \frac{d\Phi_{21}}{dt} = \frac{M_{21}}{N_2}\frac{dI_1}{dt}$$

$$\mathcal{E}_2 = -M_{21}\frac{dI_1}{dt}$$

Transformers

30-2 Self-Inductance

emf induced through self-inductance:

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

The inductance L is a proportionality constant that depends on the geometry of the circuit

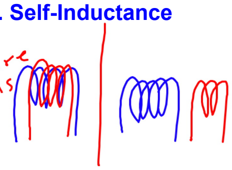
Magnetic materials will change self-inductance by changing magnetic flux

Mutual Inductance vs. Self-Inductance

Chapter 30

$$M = \frac{N_2 \Phi_{21}}{I_1}$$

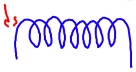
2 wire coils



$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt} = -M\frac{dI_1}{dt}$$

$$L = \frac{N\Phi_B}{I}$$

Solenoids



$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

30-3 Energy Stored in a Magnetic Field

- Work must be done to create current through inductor
- This changes the energy stored in the inductor
- Starting from zero current:

$$U_L = \frac{1}{2}LI^2$$

30-3 Energy Stored in a Magnetic Field

The energy in a solenoid depends on the current, and therefore on the magnetic field created by the current:

$$U_L = \frac{1}{2}\frac{B^2}{\mu_0}A\ell$$

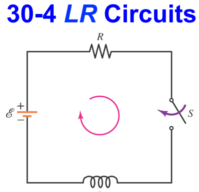
giving the energy density of the magnetic field:

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Inductance Practice (Homework)

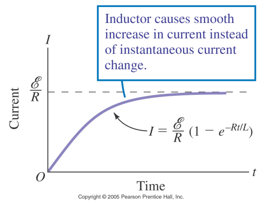
- Ex. 30-1 Solenoid and coil (§ 30-1)
- Ex. 30-2 Direction of emf in a solenoid (§ 30-2)
- Ex. 30-3 Solenoid inductance (§ 30-2)
- Energy stored in a solenoid (§ 30-3)
- Energy density in a solenoid's magnetic field (§ 30-3)
- Ex. 30-4 Coaxial cable Inductance (§ 30-2)
- Ex. 30-5 Energy stored in a coaxial cable (§ 30-3)

30-4 *LR* Circuits



When the switch closes, the inductor keeps the current from attaining its maximum value immediately. That is when the current is changing most rapidly, and when the potential drop across the conductor is at a maximum.

30-4 *LR* Circuits

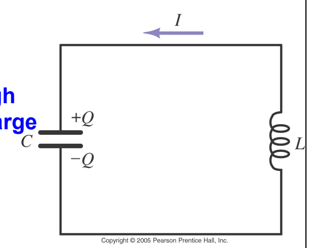


Current as a function of time:

$$I = \frac{\mathscr{E}}{R}[1 - e^{-t/(L/R)}] = \frac{\mathscr{E}}{R}(1 - e^{-Rt/L})$$

30-5 Oscillations in *LC* Circuits and Electromagnetic Oscillations

- Start with charged capacitor
- It will discharge through inductor, and then recharge in opposite sense
- If no resistance, will continue indefinitely



30-5 Oscillations in *LC* Circuits and Electromagnetic Oscillations

Charge on capacitor oscillates with frequency ω:

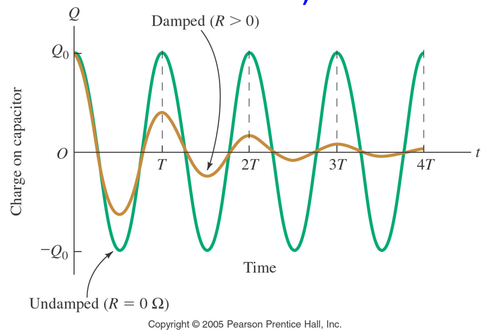
$$\omega = \frac{1}{\sqrt{LC}}$$

Charge as a function of time:

$$Q = Q_0 \cos(\omega t + \phi)$$

Here, Q_0 is the original charge and φ sets the phase at $t = 0$.

30-6 *LC* Oscillations with Resistance (*LRC* Circuits)



30-6 *LC* Oscillations with Resistance (*LRC* Circuits)

Charge equation: $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$

Solution: $Q = Q_0 e^{-\alpha t}\cos(\omega' t + \phi)$

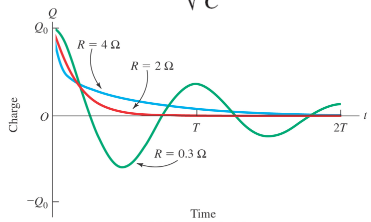
where $\alpha = \frac{R}{2L}$

and $\omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2} = \frac{1}{LC} - \alpha^2 = \omega^2 - \alpha^2$

30-6 *LC* Oscillations with Resistance (*LRC* Circuits)

For a certain value of *R*, *ω'* = 0. This is called critical damping.

$$R_c = 2\sqrt{\frac{L}{C}}$$



30-6 *LC* Oscillations with Resistance (*LRC* Circuits)

- In a pure *LC* circuit, energy is transferred back and forth between the capacitor's electric field and the inductor's magnetic field.
- Including a resistor causes I^2R losses, which show up as heat.

Homework: Inductance Practice

- Ex. 30-1 Solenoid and coil (§ 30-1)
- Ex. 30-2 Direction of emf in a solenoid (§ 30-2)
- Ex. 30-3 Solenoid inductance (§ 30-2)
- Energy stored in a solenoid (§ 30-3)
- Energy density in a solenoid's magnetic field (§ 30-3)
- Ex. 30-4 Coaxial cable Inductance (§ 30-2)
- Ex. 30-5 Energy stored in a coaxial cable (§ 30-3)

Homework: Derivations for Inductance Circuits

- *LR* Circuit (§ 30-4)
- *LC* Circuit (§ 30-5)
- *LRC* Circuit (§ 30-6)

Summary of Chapter 30

- **Definition of inductance:** $\Phi_B = LI$
- **Induced emf:** $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -L\dfrac{dI}{dt}$
- **emf induced in a second loop:** $\mathcal{E}_{21} = -M\dfrac{dI_1}{dt}$
- **Energy in an inductor:** $U_L = \dfrac{1}{2}LI^2$
- **Energy density of a magnetic field:** $u_B = \dfrac{1}{2}\dfrac{B^2}{\mu_0}$

Summary of Chapter 30, cont.

- ***LC* circuit oscillations:** $\omega = \dfrac{1}{\sqrt{LC}}$
- ***LRC* circuit oscillations:**

$$Q = Q_0 e^{-\alpha t}\cos(\omega' t + \phi)$$

30.1

$\mathcal{E}_2 = -M_{21}\dfrac{dI_1}{dt}$ $\mathcal{E}_1 = -M_{12}\dfrac{dI_2}{dt}$

$M_{21} = M_{12} = M$

$\mathcal{E}_2 = -M\dfrac{dI_1}{dt}$ $\mathcal{E}_1 = -M\dfrac{dI_2}{dt}$

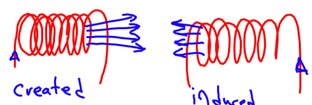
$1\ H = 1\ \dfrac{V\cdot s}{A} = 1\ \Omega\cdot s$ (Unit for inductance)

Henry

30.2

Self Inductance, L

$L = N\frac{\Phi_B}{I}$; $\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$



solenoids in circuits are inductors

30.3 Energy Stored in the B-Field

$P = IV = I\mathcal{E} = IL\frac{dI}{dt}$ $\boxed{u = \frac{1}{2}\epsilon_0 E^2}$

$\mathcal{E} = -L\frac{dI}{dt}$

$\frac{dW}{dt} = P \Rightarrow \int_0^W dW = P\,dt = \int_0^I IL\frac{dI}{\cancel{dt}}\cancel{dt} = L\int_0^I I\,dI$

$W = \frac{1}{2}LI^2$ $L = \mu_0\frac{N^2A}{\ell}$ $B = \frac{\mu_0 NI}{\ell}$

$W = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{\ell}\right)\left(\frac{B^2\ell^2}{\mu_0^2 N^2}\right) = \boxed{\frac{1}{2}\frac{B^2}{\mu_0}A\ell}$ $I = \frac{B\ell}{\mu_0 N}$

30.3 Energy Stored in the B-Field

$P = IV = I\mathcal{E} = IL\frac{dI}{dt}$ where $\mathcal{E} = L\frac{dI}{dt}$

Need to Know:

$\frac{dW}{dt} = P$; $L_{solenoid} = \mu_0\frac{N^2A}{\ell}$; $B = \mu_0\frac{NI}{\ell} \Rightarrow I = \frac{B\ell}{\mu_0 N}$

$\int_0^W dW = P\int_0^t dt = \int_0^I IL\frac{dI}{\cancel{dt}}\cancel{dt} = L\int_0^I I\,dI = \frac{1}{2}LI^2$

$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{\ell}\right)\left(\frac{B^2\ell^2}{\mu_0^2 N^2}\right) = \frac{1}{2}\frac{B^2}{\mu_0}Al$

u = energy density = $\frac{U}{\text{Volume}} = \frac{1}{2}\frac{B^2}{\mu_0}$ Similar to $u = \frac{1}{2}\epsilon_0 E^2$

Important Derivatives:

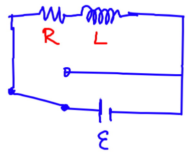
$$\frac{d}{dt}\left[Q^2\right] = 2Q\frac{dQ}{dt}$$

$$\frac{d}{dt}\left[I^2\right] = 2I\frac{dI}{dt}$$

I^2

$$\frac{dI}{dt} = 2I$$

30.4 LR Circuit



Kirchhoff's Loop Rule

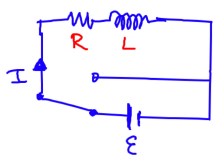
$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$\mathcal{E} - IR = L\frac{dI}{dt}$$

$$(\mathcal{E} - IR)\,dt = L\,dI \Rightarrow \frac{1}{L}\int_0^t dt = \int_0^I \frac{1}{\mathcal{E} - IR}\,dI$$

$$\frac{1}{L}t = -\frac{1}{R}\ln(\mathcal{E} - IR)\Big|_0^I = -\frac{1}{R}\ln\left(\frac{\mathcal{E} - IR}{\mathcal{E}}\right)$$

30.4 LR Circuit



$$\frac{1}{L}t = -\frac{1}{R}\ln\left(\frac{\mathcal{E} - IR}{\mathcal{E}}\right)$$

Solve for I

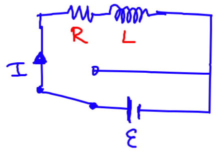
$$-\frac{R}{L}t = \ln\left(\frac{\mathcal{E} - IR}{\mathcal{E}}\right)$$

$$\frac{\mathcal{E} - IR}{\mathcal{E}} = e^{-\frac{R}{L}t} \Rightarrow \mathcal{E} - IR = \mathcal{E}e^{-\frac{R}{L}t}$$

$$\mathcal{E} - \mathcal{E}e^{-\frac{R}{L}t} = IR \Rightarrow \boxed{I = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

$\tau = \frac{L}{R}$ time constant for LR circuit

30.4 LR Circuit



$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\frac{dI}{dt} = -\frac{\mathcal{E}}{\cancel{R}}\left(-\frac{\cancel{R}}{L}\right)e^{-\frac{R}{L}t}$$

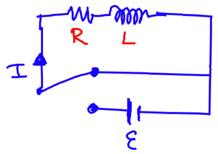
$$\frac{dI}{dt} = \frac{\mathcal{E}}{L}e^{-\frac{R}{L}t}$$

Kirchhoff's loop Rule $\quad \mathcal{E} - IR - L\frac{dI}{dt} = 0$

$$\mathcal{E} - \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)R - L\left(\frac{\mathcal{E}}{L}e^{-\frac{R}{L}t}\right) = 0$$

$$\cancel{\mathcal{E}} - \cancel{\mathcal{E}} \cancel{-\left(-\mathcal{E}e^{-\frac{R}{L}t}\right)} \cancel{-\mathcal{E}e^{-R/Lt}} = 0 \quad ✓✓$$

30.4 LR Circuit



Disconnect battery at $I = I_0 = I_{max}$

Kirchhoff's Loop Rule:

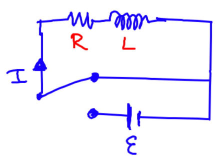
$$IR + L\frac{dI}{dt} = 0$$

$$RI = -L\frac{dI}{dt} \Rightarrow -\frac{R}{L}\int_0^t dt = \int_{I_0}^{I}\frac{1}{I}\,dI$$

$$-\frac{R}{L}t = \ln\left(\frac{I}{I_0}\right) \Rightarrow \boxed{I = I_0 e^{-\frac{R}{L}t}}$$

Verify: $\frac{dI}{dt} = -\frac{I_0 R}{L}e^{-\frac{R}{L}t}$

30.4 LR Circuit



Disconnect battery at $I = I_0 = I_{max}$

Kirchhoff's Loop Rule:

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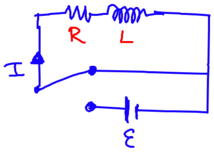
$$-\frac{R}{L}t = \ln\left(\frac{I}{I_0}\right) \Rightarrow \boxed{I = I_0 e^{-\frac{R}{L}t}}$$

Verify: $\frac{dI}{dt} = -\frac{I_0 R}{L}e^{-\frac{R}{L}t} \;\Big|\; RI_0e^{-R/Lt} - \frac{LI_0R}{L}e^{-R/Lt} = 0$

30.4 LR Circuit



Disconnect battery at

$I = I_0 = I_{max}$

Kirchhoff's Loop Rule:

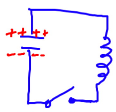
$IR + L\frac{dI}{dt} = 0$

$RI = -L\frac{dI}{dt} \Rightarrow -\frac{R}{L}\int_0^t dt = \int_{I_0}^{I} \frac{1}{I}\, dI$

$-\frac{R}{L}t = \ln\left(\frac{I}{I_0}\right) \Rightarrow \boxed{I = I_0 e^{-\frac{R}{L}t}}$

Verify: $\frac{dI}{dt} = -\frac{I_0 R}{L} e^{-\frac{R}{L}t}$ | $RI_0 e^{-R/L\, t} - \frac{L I_0 R}{L} e^{-R/L\, t} = 0$

30.5 LC Circuits



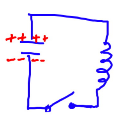
Kirchhoffs Loop Rule $[Q = VC]$

$L\frac{dI}{dt} + \frac{Q}{C} = 0$ where $I = \frac{dQ}{dt}$

$L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + \frac{Q}{C} = 0 \Rightarrow L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$

$\boxed{\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0}$ Eqn for SHM

30.5 LC Circuits



Energy

$U_{TOT} = U_L + U_C$

$= \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$

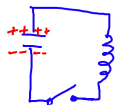
Note @ $t = 0$ $U_{TOT} = \frac{1}{2}\frac{Q_0^2}{C}$

$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}\right) = 0$ where $Q(t)$ & $I = \frac{dQ}{dt}$

$\frac{1}{2}L\left(2I\frac{dI}{dt}\right) + \frac{1}{2}\frac{1}{C}\left(2Q\frac{dQ}{dt}\right) = 0$

$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = 0 \Rightarrow L\cancel{\frac{dQ}{dt}}\left(\frac{d}{dt}\left(\frac{dQ}{dt}\right)\right) + \frac{Q}{C}\cancel{\frac{dQ}{dt}} = 0$

30.5 LC Circuits



$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0$$

$$\boxed{\frac{d^2Q}{dt^2}+\frac{Q}{LC}=0}$$ Eqn for SHM

Soln for SHM Eqn $Q=Q_0\cos(\omega' t+\phi)$

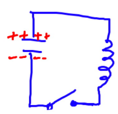
phase angle

Amplitude angular freq.

$$\frac{dQ}{dt}=-\omega' Q_0\sin(\omega' t)$$

$$\frac{d^2Q}{dt^2}=-\omega'^2 Q_0\cos(\omega' t)$$

30.5 LC Circuits



Substitute back in

$$\frac{d^2Q}{dt^2}+\frac{Q}{LC}=0$$

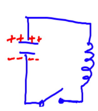
$$-\omega'^2 Q_0\cos(\omega' t)+\frac{Q_0\cos(\omega' t)}{LC}=0$$

$$\left(\frac{1}{LC}-\omega'^2\right)Q_0\cos(\omega' t)=0$$

$$\left(\frac{1}{LC}-\omega'^2\right)=0 \Rightarrow \boxed{\omega'=2\pi f=\sqrt{\frac{1}{LC}}}$$

$$\frac{dQ}{dt}=\underbrace{-\omega' Q_0}_{I_{max}=I_0}\sin(\omega' t)=I$$

30.5 LC Circuits



Substitute back in

$$U_{TOT}=\frac{1}{2}LI^2+\frac{1}{2}\frac{Q^2}{C}$$

$$\frac{1}{2}L(-\omega' Q_0\sin(\omega' t))^2+\frac{1}{2}\frac{Q_0^2\cos^2(\omega' t)}{C}$$

$$\frac{1}{2}\cancel{L}\underbrace{\omega'^2}_{\frac{1}{\cancel{L}C}}Q_0^2\sin^2(\omega' t)+\frac{1}{2}\frac{Q_0^2\cos^2(\omega' t)}{C}$$

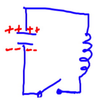
$$\frac{1}{2}\frac{Q_0^2}{C}\underbrace{\left(\sin^2(\omega' t)+\cos^2(\omega' t)\right)}_{=1}=U_{TOT}$$

$$U_{TOT}=\frac{1}{2}\frac{Q_0^2}{C} \quad \therefore U_{TOT}=\frac{1}{2}LI_0^2$$

$Q_0=Q_{max}$

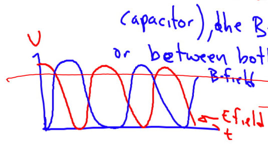
$I_0=I_{max}$

30.5 LC Circuits

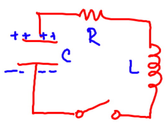


Summary of LC Circuits

The Energy stored is in either the E-field (of the capacitor), the B-field (of the inductor) or between both (it oscillates between them



30.6 LRC Circuits



Kirchhoff's Loop Rule

$$-L\frac{dI}{dt} - IR + \frac{1}{C}Q = 0$$

where $I = -\frac{dQ}{dt}$

$$-L\frac{d}{dt}\left(-\frac{dQ}{dt}\right) - \left(\frac{dQ}{dt}\right)R + \frac{1}{C}Q = 0$$

$$\boxed{L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0}$$ ← Eqn for a damped harmonic oscillator (DHO)

Soln for DHO: $Q = Q e^{-\alpha t}\cos(\omega' t)$

Conservation of Energy

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}\right) = -I^2R$$

$$\frac{1}{2}L\,2I\frac{dI}{dt} + \frac{1}{2}\frac{1}{C}2Q\frac{dQ}{dt} = -I^2R$$

$$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R \qquad \text{where } I = \frac{dQ}{dt}$$

$$L\cancel{\frac{dQ}{dt}}\frac{d}{dt}\left(\frac{dQ}{dt}\right) + \frac{Q}{C}\cancel{\frac{dQ}{dt}} = -\cancel{\frac{dQ}{dt}}\frac{dQ}{dt}R$$

$$\boxed{L\frac{d^2Q}{dt} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0}$$

$$Q = Q_o e^{-\alpha t} \cos(\omega' t)$$

$$\frac{dQ}{dt} = -\alpha Q_o e^{-\alpha t} \cos(\omega' t) - \omega' Q_o e^{-\alpha t} \sin(\omega' t)$$

$$\frac{d^2Q}{dt^2} = \alpha^2 Q_o e^{-\alpha t} \cos(\omega' t) + \alpha \omega' Q_o e^{-\alpha t} \sin(\omega' t) + \alpha \omega' Q_o e^{-\alpha t} \sin(\omega' t) - \omega'^2 Q_o e^{-\alpha t} \cos(\omega' t)$$

$$\frac{d^2Q}{dt^2} = 2\alpha\omega' Q_o e^{-\alpha t} \sin(\omega' t) + (\alpha^2 - \omega'^2) Q_o e^{-\alpha t} \cos(\omega' t)$$

Substitute above back into

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

$$L\left(2\alpha\omega' Q_o e^{-\alpha t}\sin(\omega' t) + (\alpha^2 - \omega'^2) Q_o e^{-\alpha t}\cos(\omega' t)\right) + R\left(-\omega' Q_o e^{-\alpha t}\sin(\omega' t) - \alpha Q_o e^{-\alpha t}\cos(\omega' t)\right) + \frac{1}{C}\left(Q_o e^{-\alpha t}\cos(\omega' t)\right) = 0$$

$$\boxed{(2L\alpha\omega' - R\omega') Q_o e^{-\alpha t} \sin(\omega' t) + \left(L\alpha^2 - L\omega'^2 - R\alpha + \frac{1}{C}\right) Q_o e^{-\alpha t}\cos(\omega' t) = 0}$$

$$(2L\alpha\omega' - R\omega') Q_o e^{-\alpha t} \sin(\omega' t)$$

$$\left(L\alpha^2 - L\omega'^2 - R\alpha + \frac{1}{C}\right) Q_o e^{-\alpha t}\cos(\omega' t) = 0$$

at $t = \frac{\pi}{2}\omega'$; $\cos(\omega' t) = 0$ Solve for α

$$2L\alpha\omega' - R\omega' = 0 \Rightarrow \boxed{\alpha = \frac{R}{2L}} \quad \tau = \frac{2L}{R}$$

at $t = 0$; $\sin(\omega' t) = 0$ Solve for ω'

$$L\alpha^2 - L\omega'^2 - R\alpha + \frac{1}{C} = 0$$

$$\omega'^2 = \alpha^2 - \frac{R}{L}\alpha + \frac{1}{LC} = \frac{R^2}{4L^2} - \frac{R^2}{2L^2} + \frac{1}{LC} = \frac{1}{LC} - \frac{R^2}{4L^2}$$

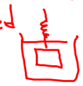
$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ $\Rightarrow \frac{1}{LC} - \frac{R^2}{4L^2} = 0 \Rightarrow R^2 = \frac{4L}{C}$

If $R = \sqrt{\frac{4L}{C}}$ critically damped doesn't oscillate

$R < \sqrt{\frac{4L}{C}}$ under damped oscillates for a long time

$R > \sqrt{\frac{4L}{C}}$ over damped



doesn't return to center for a long time

Modification of Kirchhoff's loop rule:

In moving across an inductor of inductance *L* along (or against) the presumed direction of the current *I*, the potential change is Δ*V* = –*L dI*/*dt* (or +*L dI*/*dt*, respectively). ***-el fin aside-***

Kirchhoffs Loop Rule

$-L\frac{dI}{dt} - IR + \frac{1}{C}Q = 0$ where $I = -\frac{dQ}{dt}$

$-L\left(-\frac{d^2Q}{dt^2}\right) - \left(-\frac{dQ}{dt}\right)R + \frac{1}{C}Q = 0$

$$\boxed{L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0}$$