Chapter 32

Maxwell's Equations and Electromagnetic Waves

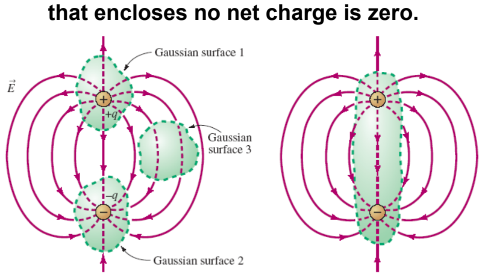
Main Points of Chapter 32

- Maxwell's Equations:
  - Gauss's Law of Electricity
  - Gauss's Law of Magnetism
  - Faraday's Law of Induction
  - Maxwell's Displacement Current and Ampère's Law
- Electromagnetic Waves and the Speed of Light

Gauss' Law of Electricity

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

The electric flux through a closed surface that encloses no net charge is zero.



Gauss' Law for Magnetism

- No magnetic monopoles (single magnetic charge) have ever been observed
- Magnetic field lines must be continuous
- The magnetic "charge" inside any closed surface must always be zero.

Gauss' Law for Magnetism

- Definition of magnetic flux:

$$\Phi_B \equiv \oint \vec{B} \cdot d\vec{A}$$

- Therefore, for any closed surface:

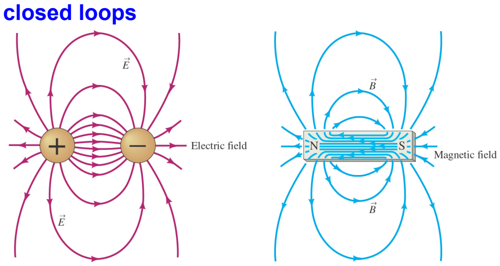
$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

This is Gauss' law for magnetism.

Gauss' Law for Magnetism

Field lines for a bar magnet:

- Similar to those for a magnetic dipole
- But no magnetic charges, so all field lines are closed loops



Faraday's Law of Induction

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

A changing magnetic flux induces an electric field; this is a generalization of Faraday's law. The electric field will exist regardless of whether there are any conductors around.

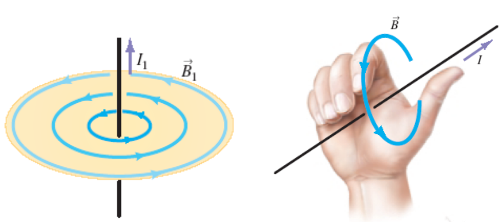
Although static electric fields are *conservative* fields, when the electric field produced by a changing magnetic field is a *nonconservative* field.

Ampère's Law

- **Experimental observation: two parallel current-carrying wires exert forces on each other**
- **Assumption: current creates a magnetic field**
- **Field can be mapped out**
- **Field makes circles around wire, direction given by right-hand rule**

Ampère's Law

Field around a current-carrying wire:



Ampère's Law

In this case, there is a current through the surface whose edge is defined by the path.

Ampère's Law is the generalization of this, valid for any current and path:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

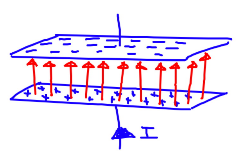
The Maxwell Displacement Current

- Ampère's law has a flaw when currents are varying
- One closed line can be the edge of an infinite number of surfaces
- As long as currents through all surfaces are the same, no problem
- But if current is varying this may not be true

Maxwell's Displacement Current

$\oint B \cdot ds = \mu_0 I_{encl}$ ← Ampère's Law (not complete)

(Note: Does not hold true for changing currents)

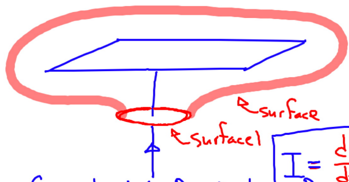


$I \Rightarrow \vec{E}$

As the capacitor charges

$I \downarrow \quad \vec{E} \uparrow$

Maxwell's Displacement Current



$\Phi_E = EA = \frac{Q}{\epsilon_0}$

$Q = \epsilon_0 EA = \epsilon_0 \Phi_E$

$$I_D = \frac{dq}{dt} = \frac{\epsilon_0 d\Phi_E}{dt}$$ Maxwell's D.C

Going back to Ampère's Law

$$\oint B\cdot ds = \mu_0 I + \mu_0 I_D = \mu_0 (I_c + I_D)$$

conduction current (I_c); Displacement current (I_D)

Ampère-Maxwell Eqn (Generalized Ampère's Law)

$$\oint B\cdot ds = \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Valid under all conditions

$\mu_0 I_c$: only applies if I is constant

$\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$: only applies when there is a changing electric flux

or $\oint B\cdot ds = \mu_0 I_c$ or $\oint B\cdot ds = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Result of Maxwell's Displacement Current

A changing electric flux produces a magnetic field!!! $\oint B\cdot ds = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Maxwell's Eqns

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ Gauss's Law for Electricity

$\oint B \cdot d\vec{A} = 0$ *Gauss's Law for Magnetism *(no magnetic monopole's)

$\oint E \cdot d\ell = -\frac{d\Phi_B}{dt}$ Faraday's Law

$\oint B \cdot d\ell = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ Generalized Ampère's Law

Maxwell's Eqns

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Note: (1) A changing magnetic field creates an electric field (Faraday's Law of Induction); (2) A changing electric field creates a magnetic field (Ampère's Law: Displacement Current)

Maxwell's Equations

I. Gauss' law for electric fields: equivalent to Coulomb's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

II. Gauss' law for magnetic fields: no magnetic monopoles

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Maxwell's Equations

III. Generalized Ampère's law: changing electric flux creates magnetic field

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

IV. Faraday's law: changing magnetic flux creates electric field

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Electromagnetic Waves

- **Electric and magnetic fields are coupled through Ampère's and Faraday's laws**
- **Once created they can continue to propagate without further input**
- **Only accelerating charges will create electromagnetic waves**

Electromagnetic Waves

Using Maxwell's equations to find an equation for the electric field:

$$\frac{\partial^2 E_x(z,t)}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x(z,t)}{\partial t^2}$$

This is a wave equation, with solution:

$$E_x = E_0 \cos(kz - \omega t + \phi)$$

And propagation speed:

$$v^2 = \frac{1}{\mu_0 \varepsilon_0} = (3.00 \times 10^8 \text{ m/s})^2$$

Electromagnetic Waves

This is the speed of light, *c*! $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

The magnetic field obeys the same wave equation.

Electromagnetic Waves

The amplitude of the magnetic field is related to the amplitude of the electric field:

$$E = cB$$

Also, the two fields are everywhere orthogonal:

$$\vec{E} \cdot \vec{B} = 0$$

Electromagnetic Waves

- Electromagnetic waves are transverse – the E and B fields are perpendicular to the direction of propagation
- The E and B fields are in phase

Electromagnetic Radiation

Maxwell's Eqn's also allowed him to calculate the speed of "light" (aka. Electromagnetic Radiation), c

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{(4\pi \times 10^{-7} \frac{T \cdot m}{A})(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2})}}$$

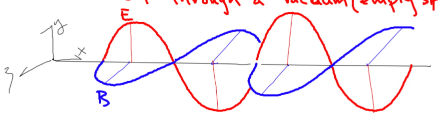
$$v = c = 2.998 \times 10^{8}\ m/s$$

Electromagnetic Radiation

because $\frac{d\Phi_B}{dt}$ produces E &

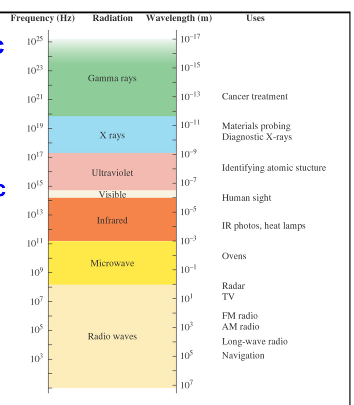
$\frac{d\Phi_E}{dt}$ produces B

Electromagnetic Radiation can propagate itself through a vacuum (empty space)



Electromagnetic Waves

The electromagnetic spectrum:



Electromagnetic Waves

• Electromagnetic waves travel more slowly through a medium by a factor *n*:

$$v = \sqrt{\frac{1}{\mu\varepsilon}} \equiv \frac{c}{n}$$

• This defines *n*, the index of refraction.

Electromagnetic Waves

• Except for ferromagnets, the speed can be written:

$$v = \sqrt{\frac{1}{\mu_0\varepsilon_0\kappa}} = \frac{c}{\sqrt{\kappa}}$$

So: $n = \sqrt{\kappa}$

The Maxwell Displacement Current

• This can be fixed by adding a term called the displacement current

• Displacement current is zero unless there is a changing electric field

$$I_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt}$$

Generalized form of Ampère's law:

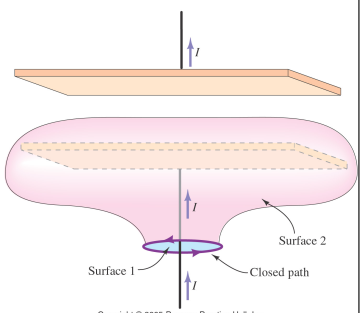
$$\oint \vec{B} \cdot d\vec{s} = \mu_0(I + I_d) = \mu_0 I + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

The Maxwell Displacement Current

For example, a capacitor charging up:

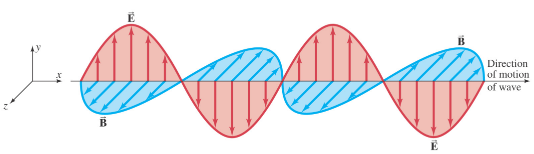
Surface 1 has a current going through it but Surface 2 does not

But: electric flux is changing through Surface 2



Chapter 32

Electromagnetic Waves



Units of Chapter 32

- Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations
- Production of Electromagnetic Waves
- Light as an Electromagnetic Wave and the Electromagnetic Spectrum
- Measuring the Speed of Light
- Energy in EM Waves
- Momentum Transfer and Radiation Pressure
- Radio and Television; Wireless Communication

32.1 Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations

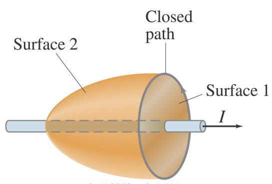
Maxwell's equations are the basic equations of electromagnetism. They involve calculus; here is a summary:

1. Gauss's law relates electric field to charge
2. A law stating there are no magnetic "charges"
3. A changing electric field produces a magnetic field
4. A magnetic field is produced by an electric current, and also by a changing electric field

32.1 Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations

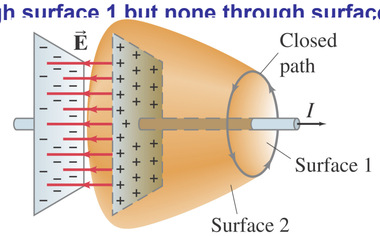
Only one part of this is new – that a changing electric field produces a magnetic field.

Ampère's law relates the magnetic field around a current to the current through a surface.



32.1 Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations

In order for Ampère's law to hold, it can't matter which surface we choose. But look at a discharging capacitor; there is a current through surface 1 but none through surface 2:



32.1 Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations

Therefore, Ampère's law is modified to include the creation of a magnetic field by a changing electric field – the field between the plates of the capacitor in this example.

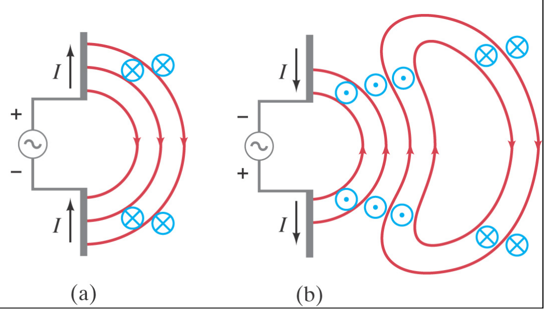
32.2 Production of Electromagnetic Waves

Since a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field, once sinusoidal fields are created they can propagate on their own.

These propagating fields are called electromagnetic waves.

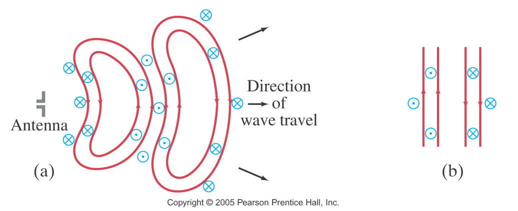
32.2 Production of Electromagnetic Waves

Oscillating charges will produce electromagnetic waves:



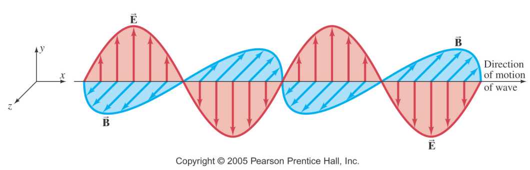
32.2 Production of Electromagnetic Waves

Far from the source, the waves are plane waves:



32.2 Production of Electromagnetic Waves

The electric and magnetic waves are perpendicular to each other, and to the direction of propagation.



32.2 Production of Electromagnetic Waves

When Maxwell calculated the speed of propagation of electromagnetic waves, he found:

This is the speed of light in a vacuum.

32.3 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

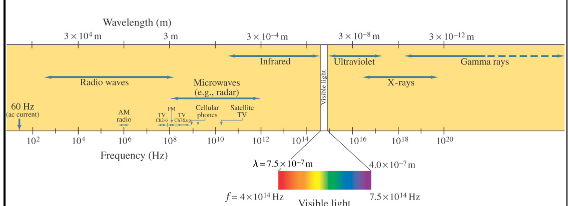
Light was known to be a wave; after producing electromagnetic waves of other frequencies, it was known to be an electromagnetic wave as well.

The frequency of an electromagnetic wave is related to its wavelength:

$$c = \lambda f \quad (32\text{-}4)$$

32.3 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

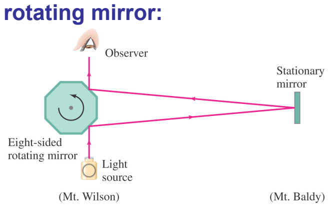
Electromagnetic waves can have any wavelength; we have given different names to different parts of the wavelength spectrum.



32.4 Measuring the Speed of Light

The speed of light was known to be very large, although careful studies of the orbits of Jupiter’s moons showed that it is finite.

One important measurement, by Michelson, used a rotating mirror:



32.4 Measuring the Speed of Light

Over the years, measurements have become more and more precise; now the speed of light is defined to be:

$$c = 2.99792458 \times 10^8 \text{ m/s}$$

This is then used to define the meter.

32.5 Energy in EM Waves

Energy is stored in both electric and magnetic fields, giving the total energy density of an electromagnetic wave:

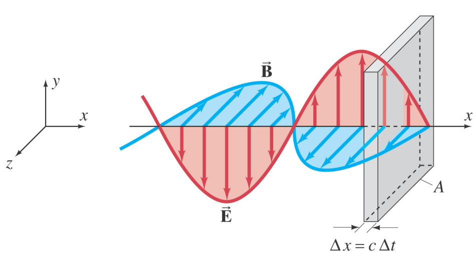
$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} \qquad \text{(32-5)}$$

Each field contributes half the total energy density.

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0 \mu_0 E^2}{\mu_0} = \epsilon_0 E^2 \qquad \text{(32-6a)}$$

32.5 Energy in EM Waves

This energy is transported by the wave.



32.5 Energy in EM Waves

The energy transported through a unit area per unit time is called the intensity:

$$I = \frac{\Delta U}{A\,\Delta t} = \frac{(\epsilon_0 E^2)(Ac\,\Delta t)}{A\,\Delta t} = \epsilon_0 c E^2 \qquad (32\text{-}7)$$

Its average value is given by:

$$\bar{I} = \frac{E_{rms} B_{rms}}{\mu_0}$$

32.6 Momentum Transfer and Radiation Pressure

In addition to carrying energy, electromagnetic waves also carry momentum. This means that a force will be exerted by the wave.

The radiation pressure is related to the average intensity. It is a minimum if the wave is fully absorbed:

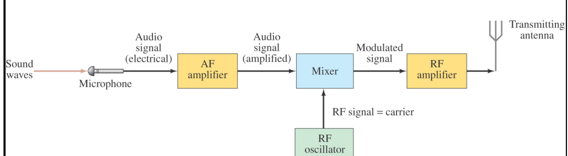
$$P = \frac{\bar{I}}{c}$$

And a maximum if it is fully reflected:

$$P = \frac{2\bar{I}}{c}$$

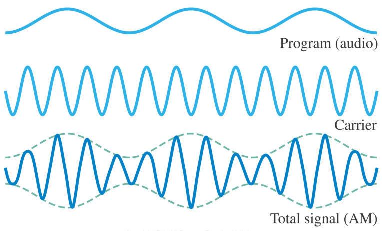
32.7 Radio and Television; Wireless Communication

This figure illustrates the process by which a radio station transmits information. The audio signal is combined with a carrier wave:



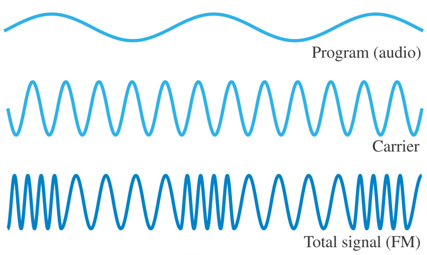
32.7 Radio and Television; Wireless Communication

The mixing of signal and carrier can be done two ways. First, by using the signal to modify the amplitude of the carrier (AM):



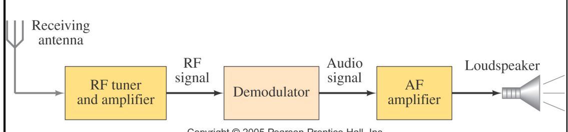
32.7 Radio and Television; Wireless Communication

Second, by using the signal to modify the frequency of the carrier (FM):



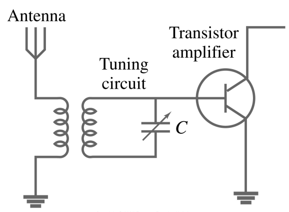
32.7 Radio and Television; Wireless Communication

At the receiving end, the wave is received, demodulated, amplified, and sent to a loudspeaker:



32.7 Radio and Television; Wireless Communication

The receiving antenna is bathed in waves of many frequencies; a tuner is used to select the desired one:



Summary of Chapter 32

- Maxwell's equations are the basic equations of electromagnetism
- Electromagnetic waves are produced by accelerating charges; the propagation speed is given by:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

- The fields are perpendicular to each other and to the direction of propagation.

Summary of Chapter 32

- The wavelength and frequency of EM waves are related:

$$c = \lambda f$$

- The electromagnetic spectrum includes all wavelengths, from radio waves through visible light to gamma rays.